Math 45 SSM 2/e 4.1 Solving Systems of Linear Equations by Graphing

Objectives: 1) Determine if an ordered pair is a solution of a system of equations.

2) Approximate the solution of a system by graphing (check by substitution)

3) Classify systems of linear equations

- consistent and independent
  - 2 lines intersect in one solution
- inconsistent
  - 2 parallel lines never intersect → no solution
- consistent and dependent
  - 2 equations are the same line → infinitely many solutions. [Every point on the line is a solution].

① Determine if $(3, 2)$ is a solution of the system

$$\left.\begin{aligned} x+3y&=9 \\ 4x-2y&=8 \end{aligned}\right\}$$

A system of equations is two or more equations to be solved simultaneously (at the same time).

A solution of a system of equations is an ordered pair (x_1, y_1) which makes both equations true.

To see if $(3, 2)$ is a solution of the system, we must check both equations.

step 1: $(3, 2)$ into $x+3y=9$ gives $3+3(2)=9$

$3+6=9$ ✓

$(3, 2)$ into $4x-2y=8$ gives $4(3)-2(2)=8$

$12-4=8$

$8=8$ ✓

$(3, 2)$ is a solution of the system ⇒ **YES**.

② Graph the system $\left.\begin{aligned} x+3y&=9 \\ 4x-2y&=8 \end{aligned}\right\}$

Call the first equation A:

$$x+3y=9$$

$$\frac{3y}{3}=\frac{-x}{3}+\frac{9}{3}$$

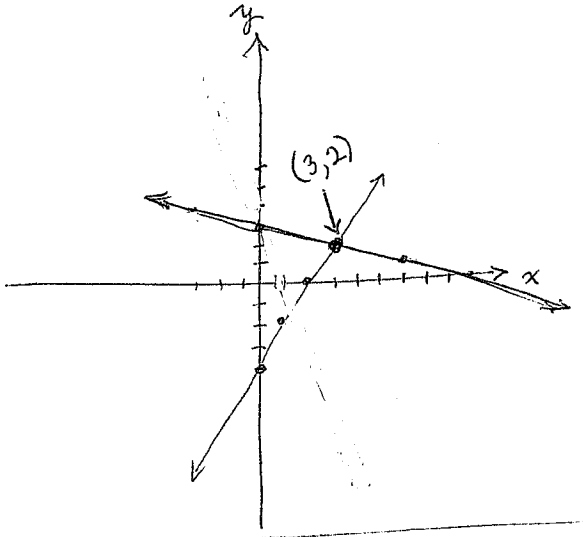
$$y=-\frac{1}{3}x+3$$

A has slope $-\frac{1}{3}$ and y-int 3

Call the second equation B

$$4x - 2y = 8$$
$$-4x \qquad -4x$$
$$-2y = -4x + 8$$
$$\frac{-2y}{-2} = \frac{-4x}{-2} + \frac{8}{-2}$$
$$y = 2x - 4$$

B has slope 2 and y-int -4



(3) Where is the solution (3,2) on the graph we drew in (2)

The solution of the system $\left.\begin{matrix} x+3y=9 \\ 4x-2y=8 \end{matrix}\right\}$ is the point where the two lines intersect, at (3,2).

(4) Determine if (1,4) is a solution of $\left.\begin{matrix} 3x+y=7 \\ -2x+3y=-12 \end{matrix}\right\}$

Substitute (1,4) into $3x+y=7$ $\quad 3(1)+4=7$, $7=7$ ✓

Substitute (1,4) into $-2x+3y=-12$ $\quad -2(1)+3(4)=-12$, $-2+12=-12$, $10=-12$ ×

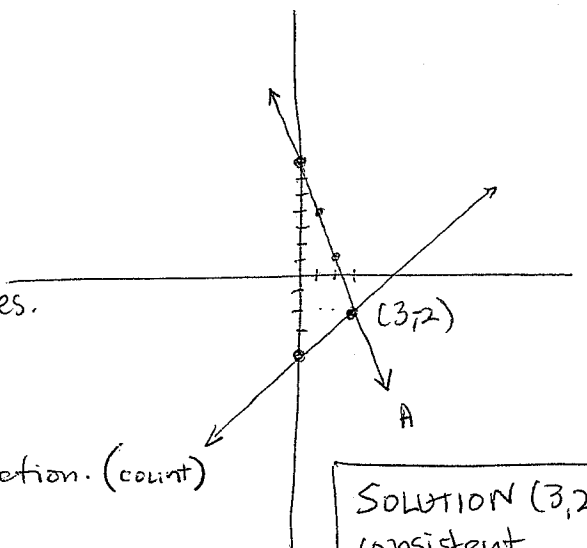
NO (1,4) is a solution of $3x+y=7$
BUT (1,4) is not a solution of $-2x+3y=-12$.
It must be a solution of both equations to be a solution of the system.

Solve the systems by graphing, classify the system.

(5) $\left.\begin{matrix} 3x+y=7 \\ -2x+3y=-12 \end{matrix}\right\}$ A, B

step 1: Graph A: $3x+y=7$
$$y = -3x+7$$

step 2: Graph B: $-2x+3y=-12$ on the same axes.
$$\frac{3y}{3} = \frac{2x}{3} - \frac{12}{3}$$
$$y = \frac{2}{3}x - 4$$



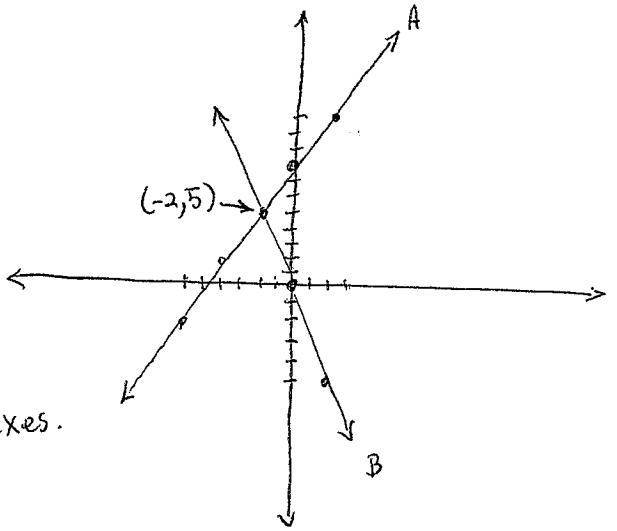
step 3: Find the coordinates of the point of intersection. (count)

step 4: Check in both equations: $3(3)+-2=7$ ✓
$-2(3)+3(2)=-12$ ✓

SOLUTION (3,2)
consistent
independent

CAUTION: You must graph very neatly for this method to work.
- Use graph paper.
- Use a ruler.
- Practice MXL problems on paper, not just the screen.
- If you don't have graph paper, make sure your tick marks on both axes are evenly spaced.
- ~~Extend accurately to edge of grid using slope.~~

if time ⑥ $\begin{cases} 3x-2y=-16 & (A) \\ 5x+2y=0 & (B) \end{cases}$



step 1: Graph $3x-2y=-16$ (A)

$$\frac{-2y}{-2} = \frac{-3x}{-2} - \frac{16}{-2}$$

$$y = \frac{3}{2}x + 8$$

step 2: Graph $5x+2y=0$ (B) on the same axes.

$$\frac{2y}{2} = \frac{-5x}{2}$$

$$y = \frac{-5}{2}x + 0$$

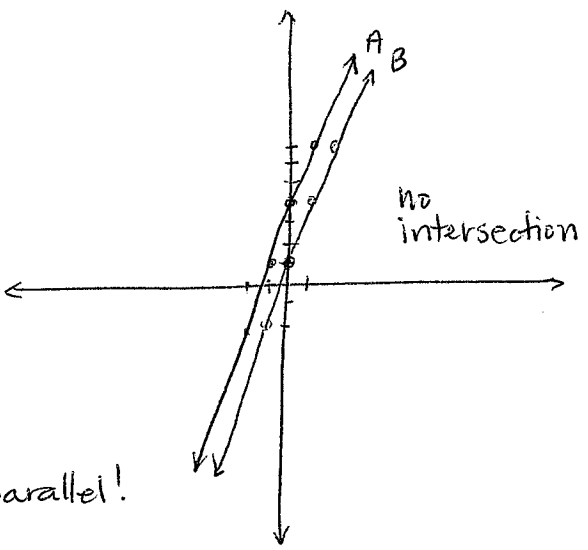
step 3 Point of intersection appears to be $(-2,5)$.

step 4: Check $3(-2)-2(5)=-16$ ✓

$5(-2)+2(5)=0$ ✓

SOLUTION $(-2,5)$
consistent
independent

yes ⑦ $\begin{cases} 3x-y=-4 & \leftarrow A \\ -6x+2y=2 & \leftarrow B \end{cases}$



step 1: Graph $3x-y=-4$

$$-y = -3x-4$$

$$y = 3x+4$$

step 2: Graph $-6x+2y=2$

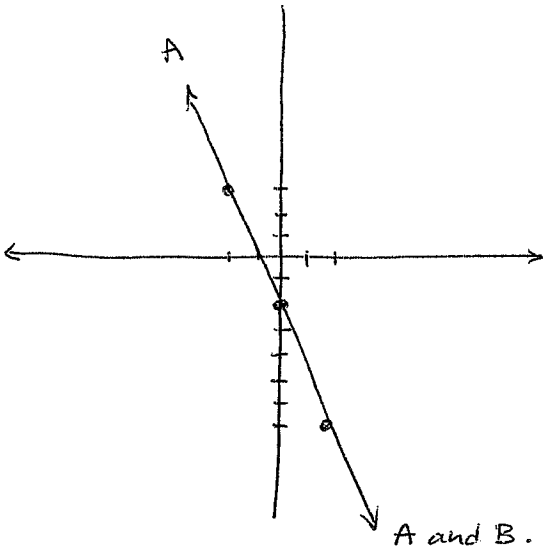
$$\frac{2y}{2} = \frac{+6}{2}x + \frac{2}{2}$$

$$y = 3x+1$$

Same slope, different y-intercepts ⇒ parallel!

NO SOLUTION
Inconsistent

yes (8) $\begin{cases} 5x + 2y = -4 & \leftarrow A \\ 10x + 4y = -8 & \leftarrow B \end{cases}$



Step 1: Graph $5x + 2y = -4$

$$\frac{2y}{2} = \frac{-5}{2}x - \frac{4}{2}$$

$$y = \frac{-5}{2}x - 2$$

Step 2: Graph $10x + 4y = -8$

$$\frac{4y}{4} = \frac{-10x}{4} - \frac{8}{4}$$

$$y = \frac{-5}{2}x - 2$$

SAME LINE!

Every point on the line is a solution.
Consistent Dependent

CAUTION: Every point on the line can be written as "infinitely many solutions". It cannot be written as "all real numbers", because there are ordered pairs that are not solutions.

Note: We can classify systems without graphing if we:

Step 1: Write each equation in $y = mx + b$ form.

Step 2: Find the slope and y-intercept of each line.

Step 3: Ask "are the slopes the same?"

if no ⇒ it's consistent independent
has one solution
(lines intersect)

if yes ⇒ ask "are the y-intercepts the same?"

if yes ⇒ it's consistent dependent
all pts on line are solutions
(same line)

if no ⇒ it's inconsistent
no solution
(parallel lines).

Classify each system without graphing.

(9) $\left.\begin{aligned} 12x+4y &= -16 \\ -9x-3y &= 3 \end{aligned}\right\}$ ← A, ← B

step 1: Write A in $y=mx+b$ form

$12x+4y=-16$

$\frac{4y}{4} = \frac{-12x}{4} - \frac{16}{4}$

$y = -3x-4$ $\quad m=-3 \quad b=-4$

step 2: Write B in $y=mx+b$ form

$-9x-3y=3$

$\frac{-3y}{-3} = \frac{9x}{-3} + \frac{3}{-3}$

$y = -3x-1$ $\quad m=-3 \quad b=-1$

step 3: Are the slopes equal? yes
Are y-ints equal? no.

inconsistent
no solution

(10) $\left.\begin{aligned} 5x+y &= -1 \\ 10x+2y &= -2 \end{aligned}\right\}$ ← A, ← B

step 1 $\quad 5x+y=-1$

$y=-5x-1$ $\quad m=-5 \quad b=-1$

step 2 $\quad 10x+2y=-2$

$\frac{2y}{2} = \frac{-10x}{2} - \frac{2}{2}$

$y=-5x-1$ $\quad m=-5 \quad b=-1$

same slopes, same y-ints.

consistent dependent
all pts on line are solutions

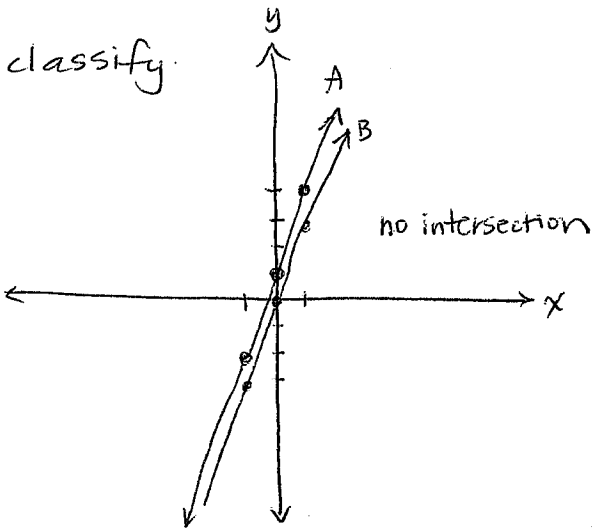
(11) $\left.\begin{aligned} y &= 2x+3 \\ y &= -2x+3 \end{aligned}\right\}$ ← A, ← B $\quad m=2 \quad b=3$; $\quad m=-2 \quad b=3$

slopes different ⇒ consistent independent
one solution

same y-int means solution is $(0,3)$.

Extra Practice: Solve by graphing, classify

(12) $\begin{cases} 3x - y = -1 & \leftarrow A \\ y = -3x & \leftarrow B \end{cases}$



step 1: $3x - y = -1$

$\frac{-y}{-1} = \frac{-3x}{-1} - \frac{1}{-1}$

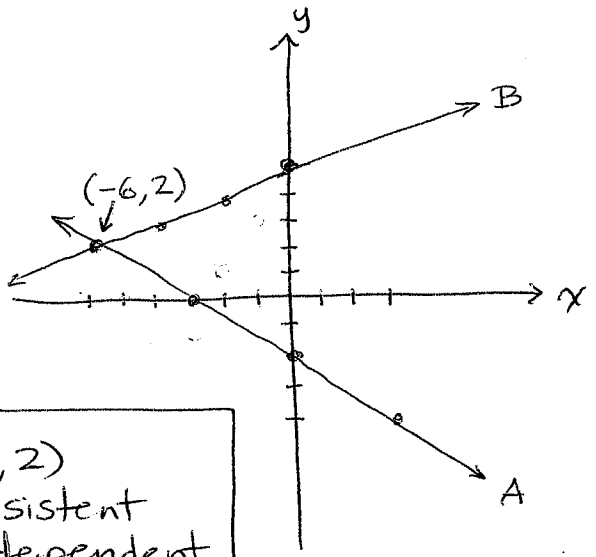
$y = 3x + 1$

step 2: $y = -3x$

parallel $\Rightarrow$ same slope, different y-int

no solution
inconsistent

(13) $\begin{cases} y = -\frac{2}{3}x - 2 & A \\ y = \frac{1}{2}x + 5 & B \end{cases}$



check

$2 = -\frac{2}{3}(-6) - 2$

$2 = 4 - 2$ ✓ A

$2 = \frac{1}{2}(-6) + 5$

$2 = -3 + 5$ ✓ B.

(-6, 2)
consistent
independent

(14) $\begin{cases} x - 4 = 0 & \leftarrow A \\ 3x - 5y = 22 & \leftarrow B \end{cases}$

A: $x - 4 = 0$

$x = 4$

Vertical

B: $3x - 5y = 22$

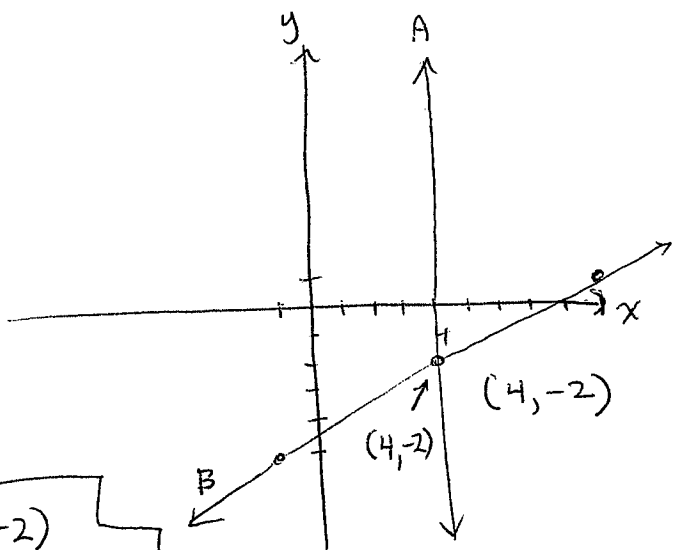
$\frac{-5y}{-5} = \frac{-3x}{-5} + \frac{22}{-5}$

$y = \frac{3}{5}x - \frac{22}{5}$ Arg!

let $y = 1$ $3x - 5 = 22$

$3x = 27$

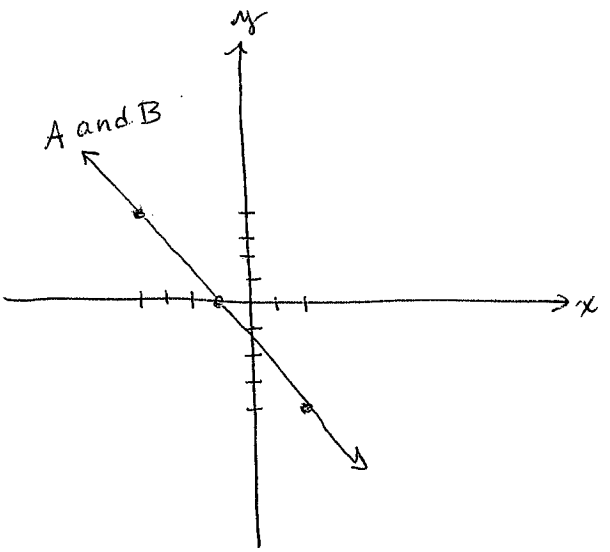
$x = 9$.



(4, -2)
consistent
independent

Extra Practice continued. Solve by graphing and classify.

(15) $\begin{cases} 3y = -4x - 4 & \leftarrow A \\ 8x + 2 = -6y - 6 & \leftarrow B \end{cases}$



step A: $\frac{3y}{3} = \frac{-4}{3}x - \frac{4}{3}$

$y = -\frac{4}{3}x - \frac{4}{3}$ arg.

Let $x = 2$

$3y = -4(2) - 4$

$\frac{3y}{3} = \frac{-12}{3}$

$y = -4$ $(2, -4)$

step B: $8x + 2 = -6y - 6$

$-6y - 6 = 8x + 2$

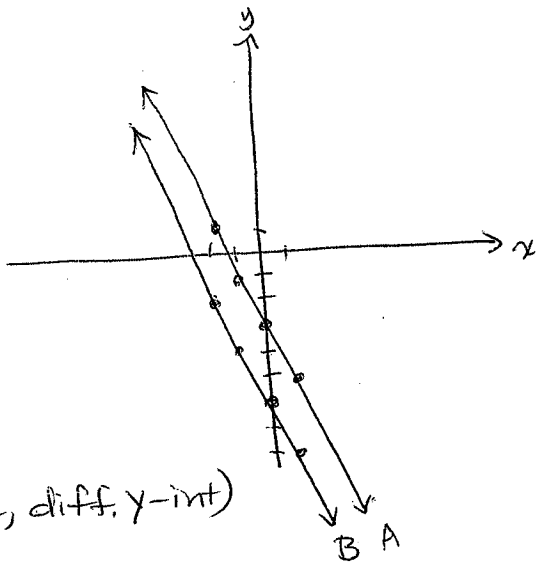
$+6 \quad +6$

$\frac{-6y}{-6} = \frac{8x + 8}{-6}$

$y = -\frac{4}{3}x - \frac{4}{3}$. same line as A!

all points on the line are solutions
consistent dependent

(16) $\begin{cases} 2x + y = -3 & \leftarrow A \\ -2x - y = 6 & \leftarrow B \end{cases}$



step A: $2x + y = -3$

$y = -2x - 3$

step B: $-2x - y = 6$

$\frac{-y}{-1} = \frac{2x}{-1} + \frac{6}{-1}$

$y = -2x - 6$

parallel lines (same slope, diff. y-int)

no solution
inconsistent

(17) A company offers two phone plans.

"Just call" plan charges $3 per month plus $0.03 per minute.

"Value plus" plan charges no monthly fee and $0.05 per minute.

a) Write a system of equations where x = number of minutes and y = costs.

b) Solve the system of equations by graphing.

a) "Just call" plan $y = 3 + .03x$ or $y = .03x + 3$ or $y = \frac{3}{100}x + 3$.

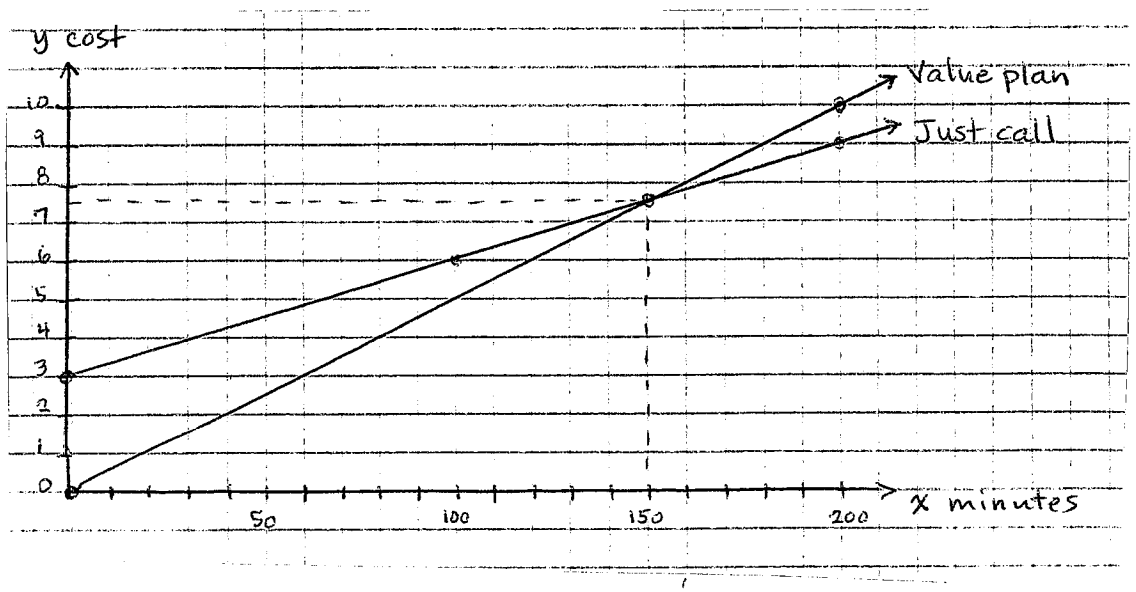
"Value plus" plan $y = 0 + .05x$ or $y = .05x$ or $y = \frac{5}{100}x$ or $y = \frac{1}{20}x$

system $\begin{cases} y = 3 + .03x \\ y = .05x \end{cases}$

or $\begin{cases} y = .03x + 3 \\ y = .05x \end{cases}$

or $\begin{cases} y = \frac{3}{100}x + 3 \\ y = \frac{1}{20}x \end{cases}$

b)



Value

x	y
0	0
100	5
200	10

Call

x	y
0	3
100	6
200	9

check (150, 7.5)? $7.5 = 3 + .03(150)$ ✓

$7.5 = .05(150)$ ✓

Solution: at 150 minutes, both plans cost $7.50